Preliminary Analysis of Small-body Gravity Estimation in the Hayabusa-2 Mission

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The Japanese asteroid explorer Hayabusa-2 will be launched in the mid-2010s to return samples from C-type near earth asteroid 1999JU3. During the rendezvous phase (i.e., proximity operation phase), we will make scientific observations to estimate physical parameters (e.g., mass, shape, pole direction, spin-rate, ephemeris) of the target body, which are very important not only for its scientific investigation but also for the spacecraft navigation. In particular, the mass is essential to perform a stable touch down sequence to collect samples from the asteroid’s surface. We will attempt to estimate the gravity field of the target body using earth-based radiometric tracking measurements (2way Doppler and range) and spacecraft-based measurements (information from optical navigation camera and laser altimetry) with global parameter estimation technique. As the first step for the gravity field estimation, we performed a simulation study about mass estimation under simple configuration and evaluated the relation between the quality and quantity of measurements and the accuracies of estimation results. The detectability of the low degree term of the gravity field coefficients is also mentioned.

Nomenclature

\( A_{sc} \) = effective cross-section area of the spacecraft, m²
\( Cr \) = solar radiation pressure coefficient
\( GM \) = gravitational parameter (gravity constant times mass), km³/s²
\( HP\_ONC_X \) = direction of the s/c in the x direction measured in the home-position coordinate system, deg
\( HP\_ONC_Y \) = direction of the s/c in the y direction measured in the home-position coordinate system, deg
\( HP\_GCPNAV \) = relative position vector of the s/c in the home-position coordinate system (calculated by picture matching method), km
\( HP\_Doppler \) = velocity of the s/c in the z direction measured in the home-position coordinate system, km/s
\( HP\_Range \) = distance between the s/c and the asteroid’s center of mass (calculated from 2way range), km
\( HP\_Lidar \) = distance between the s/c and the asteroid’s center of mass (calculated from laser altimeter), km
\( M_{sc} \) = mass of the spacecraft, kg
\( R_o \) = mean radius, km
\( \lambda_{ecl} \) = ecliptic longitude of the spin vector of the asteroid, deg
\( \beta_{ecl} \) = ecliptic latitude of the spin vector of the asteroid, deg
I. Introduction

Recently, the interest in small celestial body exploration missions that target to asteroids and comets has increased. It is expected that small bodies are primitive bodies, and provide us significant information on the solar system formation process. Although a number of inherent difficulties must be considered for missions that exploring small bodies, compared to missions to the other planets and moons in the solar system, missions to small celestial bodies provide us not only scientific knowledge but also important information on collisions of near Earth objects (NEO’s) to the Earth. The Japan’s next asteroid explore Hayabusa-2 is now under development that aims to return the asteroid samples from C-type near earth asteroid 1999JU3. This is a subsequent mission of Hayabusa that have accomplished to return samples from the asteroid Itokawa in 2010. The Hayabusa-2 spacecraft will be launched in the mid-2010 and after its cruise phase, spacecraft will rendezvous with asteroid 1999JU3 in late 2010’s. The mission phase lasts over one year and the spacecraft come back to the Earth in 2020[1].

From the point of view of the gravity estimation, before the Hayabusa mission, the Near Earth Asteroid Rendezvous (NEAR) mission (NASA/JPL, JHU/APL) target to asteroid Eros is the only case to investigate a small celestial body. In the case of NEAR mission, the spacecraft was never orbiting around the asteroid for the scientific observations, and a unique method was used for the gravity estimation, which is different from the case of NEAR mission. Recently, the Dawn (NASA) spacecraft targets to Vesta and Ceres have been launched and orbit around the asteroid Vesta at the moment. In the case of Dawn mission, the spacecraft is put into orbit in the same way as NEAR mission and the conventional 2-way RARR and optical navigation data are available for the gravity field estimation[3].

In the Hayabusa-2 mission, we are going to use a hovering approach for the scientific observation. Although the gravity field estimation is more difficult than the case of NEAR and Dawn, we would like to obtain the information about actual gravity field via the orbit determination analysis. In this study, we investigate the expected accuracy of the gravity estimation by using radiometric, optical, and altimetry measurements. In particular, the error of mass estimation, and detectability of the low degree term of the gravity coefficients are evaluated.

II. Information about the target asteroid 1999JU3

The orbit of the asteroid 1999JU3 is described in Fig. 1 with that of asteroid Itokawa as a target for comparison[4]. The perihelion is located at near the Earth orbit and the aphelion is located at near the Mars orbit.

As the results of scientific observations from ground and space telescope, several kinds of physical parameters of 1999JU3 have been derived (Table 1)[5]. At this time, two candidates for the pole direction exist, one is Kawakami’s model and the other is Muller’s model. The direction of the pole is $\lambda_{ ecl} = 331.0^{\circ}$, $\beta_{ ecl} = 20.0^{\circ}$ for the former model, and $\lambda_{ ecl} = 73.0^{\circ}$, $\beta_{ ecl} = -62.0^{\circ}$ for the later model. In this analysis, we use Muller’s model for the pole direction to transform a position of the spacecraft from the inertia coordinate system to the asteroid body-fixed coordinate system, and vice versa.

![Fig. 1 Orbit of 1999JU3 and Itokawa](Image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective diameter</td>
<td>$0.87 \pm 0.03$</td>
<td>km</td>
</tr>
<tr>
<td>$\lambda_{ ecl}$</td>
<td>73</td>
<td>deg</td>
</tr>
<tr>
<td>$\beta_{ ecl}$</td>
<td>-62</td>
<td>deg</td>
</tr>
<tr>
<td>Rotation period</td>
<td>$7.63 \pm 0.01$</td>
<td>h</td>
</tr>
<tr>
<td>Density</td>
<td>$0.5 \sim 4.0$ (C-type)</td>
<td>g/cc</td>
</tr>
<tr>
<td>Volume</td>
<td>$3.4 \times 10^8 \pm 10%$</td>
<td>m$^3$</td>
</tr>
<tr>
<td>Mass</td>
<td>$1.7 \sim 14.0 \times 10^{11}$</td>
<td>kg</td>
</tr>
<tr>
<td>GM</td>
<td>$11 \sim 91$</td>
<td>m$^{3}$/s$^2$</td>
</tr>
<tr>
<td>Resonance radius</td>
<td>$0.601$ (GM=11)</td>
<td>km</td>
</tr>
<tr>
<td></td>
<td>$0.996$ (GM=51)</td>
<td>km</td>
</tr>
<tr>
<td></td>
<td>$1.201$ (GM=91)</td>
<td>km</td>
</tr>
</tbody>
</table>

Table 1 Physical parameters of 1999JU3
III. Simulation analysis for mass estimation

A. Overview

The first step of the actual gravity field estimation is to evaluate the mass of the target asteroid. The knowledge of the mass is invaluable information not only for scientific analysis but also for the operation of the spacecraft, especially in the vicinity of the asteroid surface. The error of mass estimation has a direct effect on the accuracy of navigation in the touch down phase.

The objective of this analysis is to evaluate the relation between estimation error and the setting for the orbit determination (e.g., motion range of the spacecraft, arc length, measurement availability, measurement accuracy). According to the Hayabusa’s experience, the fluctuations caused by the reaction control system (RCS) have harmful effects for the gravity estimation. Therefore, we suppose that the spacecraft perform almost vertical dive without RCS events (i.e. ballistic flight) and the available measurements are HP_Doppler, HP_ONC, and HP_Lidar. The HP_GCPNAV and HP_Range measurements are not included with consideration for the availability in the operation sequence at this time.

B. Method

In this analysis, we use the “home-position (HP) coordinate system” as the quasi-inertia coordinate system (Fig. 2). The origin is taken at the center of the asteroid, with the Z-axis toward to the Earth, the Y-axis normal to the plane, which is spanned by the 1999JU3-Earth vector and 1999JU3-Sun vector, and the X-axis completes the right-hand coordinate system.

The solar radiation pressure (SRP) and the gravitational acceleration of the Sun and planets are considered for the orbit calculation. In addition, the gravitational acceleration of the asteroid is also included. A cannonball model is adopt for the SRP model with assuming the following parameters: Cr = 1.3, \( A_{avc} = 13 \text{ m}^2 \), \( M_{avc} = 600 \text{ kg} \), and a simple point-mass gravity model is used for the gravitational acceleration of the asteroid.

The accelerations caused by the SRP and the gravitational acceleration of the asteroid that is calculated three different GM values, are described in Fig. 3. In the case of the GM value is 51 m\(^3\)/s\(^2\), the magnitude of SRP acceleration equals to that of gravitational acceleration at about 20 km. In other words, the estimation error of the SRP coefficient has a negative influence on gravity estimation, and we have to perform careful evaluation for the effect of SRP at the relatively high altitude phase.

The setting for simulation is summarized in Table 2. We assume that the spacecraft perform the vertical dive and the motion ranges of the spacecraft vary from 20 km (highest altitude case) to 2.5 km (lowest altitude case). The estimation parameters are: the position and velocity of the spacecraft at the epoch time, SRP coefficients, and GM of the asteroid. As to the initial position and velocity at the orbit determination, we use slightly different values.

### Table 2 Setting for simulation

<table>
<thead>
<tr>
<th>Pseudo Measurement</th>
<th>a priori</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM (m(^3)/s(^2))</td>
<td>51</td>
</tr>
<tr>
<td>Cr</td>
<td>1.3</td>
</tr>
<tr>
<td>Observables</td>
<td></td>
</tr>
<tr>
<td>HP_Doppler</td>
<td>0.2 mm/s</td>
</tr>
<tr>
<td>HP_ONC</td>
<td>0.1 deg</td>
</tr>
<tr>
<td>HP_Lidar</td>
<td>10 m</td>
</tr>
<tr>
<td>Motion time (hour)</td>
<td>1, 6, 12</td>
</tr>
<tr>
<td>Min altitude (km)</td>
<td>2.5, 4.0, 5.0, 6.5, 7.5</td>
</tr>
</tbody>
</table>

Fig. 2 Home-position coordinate system

Fig. 3 Gravitational acceleration of 1999JU3
compared with the setting to make pseudo measurements. The availability for the measurements are, 1 obs. / 1 min for \texttt{HP\_Doppler}, 1 obs. / 2 min for \texttt{HP\_ONC} and \texttt{HP\_Lidar}, respectively.

C. Results of mass estimation

The estimation errors are summarized in Fig. 4 as a function of minimum altitude of the spacecraft motion. The longer motion time and the lower terminal point seem to yield better results for mass estimation. According to these results, when we use the measurements that were taken during about 12 hours vertical descent motion, we can estimate the GM with a few percent error.

IV. Simulation analysis for gravity field estimation

A. Overview

The next interest for the actual gravity field is to estimate the low degree term of the spherical harmonics, which bring us knowledge about the information of interior structure of the asteroid. In the case of NEAR mission, over the 10th degree and order coefficients had been estimated using radiometric and optical measurements. Thus, the difference between the center of figure (COF) and the center of mass (COF) could be calculated. From the comparison of gravity analysis with shape analysis, the only small difference between COF and COF is fined, and these results indicate that the interior of the asteroid Eros is seems to be homogenous.

On the other hand, in the case of hovering approach, there are a lot of altitude control maneuvers and we have less opportunity to acquire the measurements during ballistic flight that is usable for gravity analysis. In this analysis, we evaluate the detectability of the offset between COF and COM, which is one of the most important physical parameters of target asteroid. Specifically, the relation between the number of ballistic flight pass and the estimation accuracy of the COM-COF offset is investigated.

B. Method

In the case when the spacecraft is close to the asteroid, the distribution of the asteroid’s mass should be taken into account for the calculation. As to the gravity model, the spherical harmonics gravity model and the polyhedron gravity model were used very often in small body missions (e.g., NEAR, Hayabusa, Dawn)[6][7][8]. Each model has both advantage and disadvantage. It is important to note that the error of the spherical harmonics gravity model increases when evaluating the gravity field close to the model’s radius and is no longer guaranteed to converge in the circumscribing sphere. Therefore, in order to analyze the orbit of the spacecraft in this region, we have to adopt the polyhedron gravity model. In this analysis, we assume that the spacecraft move the outside of the circumscribing sphere, so the spherical harmonics gravity model is used in orbit calculation.

The asteroid’s gravity potential is described as follows[9][10]:

\[
U = \frac{GM}{r} \sum_{e=0}^{n} \sum_{m=0}^{\min(n,e)} \frac{R_{e}}{r} \sin^2 \phi \left[ C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right] (C_{nm}, S_{nm}) = \sqrt{(n + m)^{2} - \delta_{nm}^{2}} \frac{(n + m)}{(2n + 1)(n - m)} (C_{nm}, S_{nm})
\]

Fig. 4 Estimation error of GM

Fig. 5 Latitude of Sub-Earth point

\[
C_{nm}; S_{nm} = \frac{1}{n + m} \frac{(n + m)!}{(2n + 1)(n - m)!} \quad (C_{nm}, S_{nm})
\]
Fig. 6a Orbit of hayabusa-2 for segment-1 (1999JU3 body-fixed coordinate system)

Fig. 6b Magnitude of accelerations caused by each gravity coefficient term for segment-1.

Fig. 7a Orbit of hayabusa-2 for segment-2 (1999JU3 body-fixed coordinate system)

Fig. 7b Magnitude of accelerations caused by each gravity coefficient term for segment-2.

Fig. 8a Orbit of hayabusa-2 for segment-3 (1999JU3 body-fixed coordinate system)

Fig. 8b Magnitude of accelerations caused by each gravity coefficient term for segment-3.
According to Rossi[11] and Friedlander[12], the gravity coefficients (C20, C22, C40, C42, C44) for triaxial ellipsoid are described as follows:

\[
\begin{align*}
C_{20} &= \frac{2c^2 - (a^2 + b^2)}{10R_c^2} \\
C_{22} &= \frac{a^2 - b^2}{20R_c^2} \\
C_{42} &= \frac{(a^2-b^2)(2c^2-a^2-b^2)}{280R_c^4} \\
C_{44} &= \frac{(a^2-b^2)^2}{2240R_c^8} \\
C_{40} &= 3\left(\frac{a^2 + b^2}{280R_c^2}\right) + 8\left(\frac{a^2b^2}{280R_c^2}\right) - 8\left(\frac{(a^2 + b^2)}{280R_c^2}\right)
\end{align*}
\] (2)

Then, by expanding to forth order equation (1), we obtain the following expression for the potential:

\[
U = \frac{GM}{r} \left[ 1 + \left(\frac{r}{R}\right)^2 C_{20}(3\sin^2\theta - 1) + 3C_{22}\cos^2\theta\cos(2\lambda) + \left(\frac{r}{R}\right)^4 \frac{1}{8} C_{40}(35\sin^4\theta - 30\sin^2\theta + 3) \right. \\
+ \frac{15}{2} C_{42}\cos^2\theta(7\sin^2\theta - 1)\cos(2\lambda) + 105C_{44}\cos^4\theta\cos(4\lambda) \right]
\] (3)

Setting for the simulation, the motion range of the spacecraft is from the 20 km to 1.5 km at the home-position coordinate system and the motion time is 12 hour. In addition, we assume that the each analysis case consists of a set of vertical descent and vertical ascent phase.

When we use the asteroid body-fixed coordinate system, it is important to consider not only the direction of the pole but also the descent region of the asteroid surface. According to the Muller’s model, the sub-earth point (i.e. objective point for the descent sequence) vary from about -30 degree to +30 degree with time (Fig. 5). To consider the effect of variation of sub-earth point, three different cases are set: 0 degree for segment-1, +29 degree for segment-2, -27 degree for segment-3. The trajectories of the spacecraft, and the accelerations caused by each gravity coefficients are described in Fig. 6, 7, 8.

The pass settings are as follows: Case-1 use segment-1 only, Case-2 use segment-1 and segment-2, Case-3 use segment-1, segment-2, and segment-3. The estimation parameters are: the position and velocity of the spacecraft at the epoch time, SRP coefficients (Cr), GM of the asteroid, and the low degree gravity coefficients (C_l20, C_l22, S_l11). To evaluate a detectability of COM-COF offset, we intentionally set 10 m offset in each axis for pseudo measurements. Except the position and velocity (e.g., GM, Cr, C_l10, C_l11, S_l11) are treated as the global parameters.

C. Results of gravity estimation

The estimation results of three cases are summarized in Fig. 9. The offsets between the ideal value and estimated value are: 41 m for Case-1, 11 m for Case-2, and 3 m for Case-3. According to these results, 6 passes (a segment contains 2 passes) need to estimate the COM-COF offset within the 10 m errors.

During the mission phase, there are several opportunities to obtain the measurements under ballistic flight, and we are going to combine all effective flight passes to estimate COM-COF offset.

**V. Summary**

In this study, the gravity estimation of the asteroid 1999JU3 is investigated, especially the error of the mass and low degree term of the gravity coefficients are evaluated. As to the mass estimation, we can expect to estimate a asteroid’s mass within a few percent error by using HP_Doppler, HP_ONC, HP_Lidar measurements that are acquired in about 12 hours ballistic vertical descent or vertical ascent motion.
The detectability of the offset between COF and COM is also evaluated. In the case of the hovering approach, although it is difficult to estimate the high order term of the gravity coefficients, there is a good possibility that to estimate a COM-COF offset to combine the multiple passes.

From the point of view of the orbit determination of the spacecraft, there is further possibility to improve the accuracy and consistency using altimetry crossover measurements[13].

At this time, the relatively optimistic preconditions are used in the analysis, thus we are going to update the model more specific. We will also analyze the effectiveness of the crossover observation for orbit determination and initial shape estimation.

References